Radial orbit instability : review and perspectives

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Abstract

This paper presents elements about the radial orbit instability, which occurs in spherical self-gravitating systems with a strong anisotropy in the radial velocity direction. It contains an overview on the history of radial orbit instability. We also present the symplectic method we use to explore stability of equilibrium states, directly related to the dissipation induced instability mechanism well known in theoretical mechanics and plasma physics.

Keywords : gravitation, instability, radial orbits, symplectic

1 Introduction

An interesting problem in Astrophysics is the study of N-body self-gravitating systems with a lot of radial orbits, when most particles have very prolonged orbits with near-radial velocities, and come close to the system's center. The stability of such systems remains an open question to this day, although several elements suggest out that spherically symmetric systems of radial orbits could be unstable, and susceptible to lose their initial sphericity. This is called the radial orbit instability, henceforth referred to as ROI. This mechanism could actually be an explanation for the shape of some astrophysical objects, such as elliptical galaxies, that are hard to explain otherwise (as gravity usually tends to create spherical objects).

For sufficiently large values of N, which is certainly the case for galaxies, N-body self-gravitating systems can be described by a smooth distribution function, and two-body interactions or "collisions" can be neglected in front of the whole gravitational potential created by this function.[1] For simplicity, we will assume that each body has the same mass m.

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[1]To see that, we can consider the two characteristic time constants of its dynamics. *Crossing time* is the typical time a particle takes to go across the system during its movement, it describes its trajectory in the potential created by other particles as a whole. *Relaxation time* is the time it takes for a particle's trajectory to be significantly influenced by collisions.

Combining the Vlasov equation (describing how a distribution function evolves in time in a given potential when collisions are neglected) and the Poisson equation (giving the potential created by this very function), we get the Vlasov–Poisson system :

$$\left\{\begin{array}{l} \frac{\partial f}{\partial t} + \frac{\mathbf{p}}{m} \cdot \nabla_{\mathbf{q}} f - m \nabla_{\mathbf{q}} \psi \cdot \nabla_{\mathbf{p}} f = \frac{\partial f}{\partial t} + \{f, E\} = 0 \\ \psi(\mathbf{q}) = -Gm \int \frac{f'}{|\mathbf{q}-\mathbf{q}'|} \mathrm{d}\Gamma' \end{array}\right. \tag{1}$$

Here, $\mathrm{d}\Gamma$ is a shorthand for $\mathrm{d}^3\mathbf{q}\mathrm{d}^3\mathbf{p}$, and the notation a' does not denote a derivative of a, but instead the value $a(\mathbf{q}', \mathbf{p}', t)$ (for any variable a). E is the average energy per particle $E(\mathbf{q}, \mathbf{p}, t) := \frac{\mathbf{p}^2}{2m} + m\psi(\mathbf{q})$ (or one-particle Hamiltonian).

Spherically symmetric, stationary solutions to this system can be either isotropic in velocity space, in which case the distribution function can be written as a function of the energy per particle alone, $f = f_0(E)$; or, in the case where velocity distribution is not isotropic, as a function of energy and kinetic momentum $f_0(E, L^2)$ (see for example Perez, Alimi, Aly, and Scholl, 1996).[2]

Considering the amount of work done about radial orbit instability (Roi) over the years, and the importance of this concept, we felt it was necessary to do a complete timeline of publications about it; this will be the second part of the present paper. In a third part, we will present the symplectic method that we use to study the stability of self-gravitating systems, and its prospects on the study of radial orbit instability.

2 Historical overview of radial orbit instability

2.1 The pioneers

The first important work published about Roi is an analytical result by Antonov (1973). It establishes a differential system for a given "displacement" of orbits and the corresponding Poisson equation, in the limit case of radial orbits. Instability of the system is then proved by constructing a strict Lyapunov function for this system, although the proof is unclear.

The same year, Hénon (1973) published one of the first numerical simulations of the problem, using $N = 1000$ spherical shells. Isotropic, polytropic models $f(E) \propto E^n$ were found to be stable (as it was obtained by Antonov in the 60s), whereas anisotropic systems, for generalized polytropes $f(E) \propto E^n L^{2m}$, were found to become unstable when $m \to -1$, which corresponds to a system with more and more radial orbits. The method does not allow to see the effect on the position space (the famous "bar"). This article made no reference to the work of Antonov.

A contrary result was published by the French team of Gillon, Doremus, and Baumann (1976). Using water bag methods (decomposing distribution functions

A calculation, based on the velocity change during an encounter and the typical number of encounters during one crossing, shows (see for instance Binney and Tremaine (2008), chapter 1.2.1.) that relaxation time is typically of order $\frac{N}{\ln N}$ larger than crossing time. This means that for durations comparable to the crossing time, the effect of collisions can legitimately be neglected.

[2] Conversely, a distribution function $f_0(E)$ describes indeed an isotropic, spherically symmetric equilibrium, and $f_0(E, L^2)$ a spherically symmetric, anisotropic equilibrium.

as a sum of functions constant over phase space domains),[3] they predicted that all self-gravitating systems $f(E, L^2)$ were stable against non spherical perturbations.

2.2 Subsequent advances

In the eighties, Polyachenko and Shukhman (1981) proposed a matrix formulation of the stability problem, based on a Fourier series decomposition of perturbations; this allows them to prove that models of the form $f(E - \frac{\lambda L^2}{r_a^2})$ (later called Ossipkov–Merritt models) are unstable if r_a^2 is sufficiently small. The article unfortunately isn't very clear either, but the result is contrary to Gillon et al. and confirms Antonov and Hénon. This article also presents an often-quoted stability criterion, stating that radial orbit instability occurs when $\frac{2T_r}{T_\perp} > 1.75 \pm 0.25$, T_r and $T_\perp$ being respectively the system's total radial and orthogonal kinetic energies.

The first full, realistic numerical survey of the problem of gravitational collapse was realized by van Albada (1982). This survey considers sets of $N = 5000$ particles, for different initial conditions : homogeneous spheres and systems made of smaller homogeneous spheres (clumps), for different speed distributions determined by the initial virial ratio. The results are clear : while collapsing homogeneous spheres are not affected by radial orbit instability, clumped systems with violent collapse (small virial ratio) lead to a triaxial equilibrium. The resulting light and density profiles are compatible with the ones observed on galaxies (the $r^{1/4}$ law).

One of the first complete studies of instability, both analytical and numerical, was made by Barnes, Hut, and Goodman (1986). Numerical studies with N-body methods confirm and complete Hénon's results for generalized Plummer models, as well as the Russians' stability criterion. Their analytical explanation for Roi links it to Jeans instability, stellar pressure in the tangential direction would no longer be sufficient to offset the natural tendency of radial orbits to condense.

The next paper on the subject[4] by Merritt and Aguilar (1985), focuses on numerical results. It uses N-body simulations with $N = 5 \cdot 10^3$, taking as initial conditions "galactic type" density profiles $\rho(r) \propto (r/r_0)^{-2}(1+r/r_0)^{-2}$ (the well-known Jaffe model, compatible with the density profile $r^{1/4}$). These profiles can be converted easily to Ossipkov–Merritt models (which are isotropic near the center and anisotropic towards the borders). The results are that transition between stability and instability is fairly sharp, and happens for $\frac{2T_r}{T_\perp} \approx 2.5$, a bit more than predicted by the Russian criterion. However, comparison with a distribution function decreasing in E and in L^2 seems to point out that the value of $\frac{2T_r}{T_\perp}$ isn't a reliable stability criterion. The article also puts forward (apparently for the first time) the idea that Roi can be useful in explaining galactic formation.

The English team of Palmer and Papaloizou (1987) made a purely analytical study (the first one since the alleged Russian result, disregarding the waterbag

[3] Five years earlier, Doremus, Feix and Baumann had obtained the stability of isotropic systems by the same method.

[4] It actually references Barnes et al. (1986), in spite of an apparently earlier publication date.

result of Gillon et al.) based on a spectral analysis of perturbations, decomposing them once again on a family of orthogonal functions. This study seems to indicate an instability, although is seems very hard to verify. Two other important aspects of this paper are a "demonstration" that the Russian criterion (already shaken by Merritt and Aguilar) is invalid, and a presentation of a new mechanism for instability growth, inspired by a work by Lynden-Bell (1979) : an axisymmetric perturbation of the potential in a spiral galaxy could influence a star's orbit and lengthen it, which tends to align orbits along the perturbation. This effect could play a part in bar formation in spiral galaxies.

A clear synthesis of all those results was made by Merritt (1987), including Lynden-Bell's mechanism, as well as a criticism of Jeans instability mechanism for needing an homogeneous system which is not the case here.

Katz (1991) brought in a new kind of simulations in this context, which showed that "authentic" cosmological simulations with a Hubble flow and merging phases tend to erase traces of possible primordial Roi. The same year, an article by Saha (1991) extended the reach of spectral methods for normal modes to infinite-extension models, which was not the case for previous studies. Still the same year, a study by Weinberg (1991) used the matrix methods initiated by the Russian school of Polyachenko, and found some results again. This analysis was later the subject of a complete article by the Argentine team of Cincotta, Nunez, and Muzzio (1996), who studied transformation of loop orbits into bow orbits, in the style of Lynden-Bell's mechanism and in accordance to the intuition of Merritt (1987).

2.3 A renewed interest

Taking advantage of some of their analytical results, Perez et al. (1996) proposed and tested a new stability criterion for self-gravitating systems based on the nature of the perturbations it is submitted to. This criterion is validated on Ossipkov–Merritt models applied to polytropes, the number of particles involved reaching for the first time reasonable values of $N \approx 10^4$ for the whole of simulations. In their analytical results, they explain how waterbag methods are lacking in the field of radial orbits, which may explain the now-discredited result of Gillon et al. (1976).

The German team of Theis and Spurzem (1999) undertook an extensive numerical study of Roi, using dedicated "Grape" machines for collapses of Plummer spheres with varying initial temperature. Growth rate of Roi is largely affected by potential softening, and very little by variations of the number of particles. Those simulations highlighted a very long-term evolution (its time scale is relaxation time) of triaxial systems produced by Roi towards a more or less spherical system, an evolution that according to the authors is due to collisions.

A systematic study of gravitational collapse with tests for several numerical parameters (N, softening, . . .) by Roy and Perez (2004) allowed, among other results, to stress that Roi is dependent on the presence of robust inhomogeneities in the precollapse system. Only several-scale collapses can lead to Roi : collapses of homogeneous spheres fail to do so. Those results were completed and refined by Boily and Athanassoula (2006), who showed a small effect of particle number on the final state.

Although the role of ROI in structure formation was hurt by the aforementioned work of Katz (1991), complementary analyses by the German team of Huss, Jain, and Steinmetz (1999) and by the Canadian team of MacMillan, Widrow, and Henriksen (2006) observed the result of medium-scale structure formation by collapse experiments, with the possibility of numerically suppressing ROI. Acceptable density profiles are only found when ROI actually takes place during primordial phases; otherwise structure profiles are incompatible with simulations and observations! This gives a new argument for ROI as a fundamental process of structure formation.

A new activity in this domain is rising since 2005, notably from E. Barnes's team. Their articles, notably Barnes, Williams, Babul, and Dalcanton (2005) and Bellovary, Dalcanton, Babul, Quinn, Maas, Austin, Williams, and Barnes (2008), show that ROI not only creates a triaxial system in position space (which was known for a long time), but also creates a spacial segregation in velocity space (with an isotropic center and a radial halo). This segregation could be the root of the universal profile observed in large structures. Follow-up papers, including Barnes, Lanzel, and Williams (2009), indicated that ROI is not found in constant-density collapse, a statement backed up by the Italian team of Trenti and Bertin (2006). This agrees with the previous explanation by Roy and Perez (2004), that ROI does not happen without a primordial equilibrium state caused by a several-scale inhomogeneous collapse.

Lastly, ROI has been observed in a triaxial state by Antonini, Capuzzo-Dolcetta, and Merritt (2009) : it apparently happens when this state is populated with too many "box orbits with predominantly radial motions". The system would then become more prolate, and still triaxial.

As this overview shows, ROI is a phenomenon that has been known for almost 40 years, but still sparks some controversy and contradictory results. It seems to be fundamental in structure formation, yet there are still a lot of unanswered questions on both the physical and the analytical sense. Under which conditions does it happen, if it happens? Can we explain its mechanism? Several tools can be used to study radial orbit systems, and among them, potential energy methods could prove quite useful. We chose to focus on the symplectic method, which will be covered in the next section.

3 The symplectic method

3.1 Presentation

The symplectic method is a way of investigating the stability of a steady state against possible perturbations. It was first developed by Bartholomew (1971), though its use in the study of gravitational plasmas is a relatively recent development. See for instance Kandrup (1991).

This method makes use of the Hamiltonian structure of the system under scrutiny. The Vlasov–Poisson system indeed derives from the following Hamiltonian :

$$H\left[f\right] = \int \mathrm{d}\Gamma \frac{\mathbf{p}^2}{2m} f\left(\Gamma, t\right) - \frac{1}{2} G m^2 \int \mathrm{d}\Gamma \int \mathrm{d}\Gamma' \frac{f\left(\Gamma, t\right) f\left(\Gamma', t\right)}{\left|\mathbf{q} - \mathbf{q}'\right|}$$

which is just the system's total energy. Note that its functional derivative $\frac{\delta H}{\delta f}$

is the one-particle Hamiltonian $E = \frac{\mathbf{p}^2}{2m} + m\psi$.

This structure allows us to easily compute the time variation of any functional of the distribution function. If $K[f]$ is a derivable functional of f, then by definition of $\frac{\delta K}{\delta f}$:

$$\frac{\mathrm{d}K[f]}{\mathrm{d}t} = \int \frac{\delta K}{\delta f}\frac{\partial f}{\partial t}\mathrm{d}\Gamma = \int \frac{\delta K}{\delta f}\left\{E, f\right\}\mathrm{d}\Gamma \tag{2}$$

Here, we use the noncanonical Poisson bracket, which was introduced by Morrison and Greene (1980) for studies in hydrodynamics and magnetohydrodynamics (and thus sometimes referred to as the Morrison bracket). For two functionals A and B of f, it is defined by

$$[A, B](f) := \int f\left\{\frac{\delta A}{\delta f}, \frac{\delta B}{\delta f}\right\}\mathrm{d}\Gamma$$

Then, from (2) it follows that

$$\frac{\mathrm{d}K[f]}{\mathrm{d}t} = -\int f\left\{\frac{\delta H}{\delta f}, \frac{\delta K}{\delta f}\right\}\mathrm{d}\Gamma = [K, H](f) \tag{3}$$

As shown by Kandrup (1991), all physical perturbations $f^{(1)}$ that f_0 can receive may be written in the form :

$$f^{(1)}(\Gamma, t) = -\left\{g, f_0\right\}$$

g is called the *generator* of the perturbation. We denote as G the following operator :

$$G[f] := \int fg\mathrm{d}\Gamma$$

Writing perturbations in this form allows us to compute the energy variation. A first-order calculation shows that

$$H^{(1)}[f_0] = [G, H](f_0) = -\int g\{f_0, E\}\mathrm{d}\Gamma = 0$$

Since f_0 is a steady state, $\{f, E\} = 0$ so the energy variation is zero at first order. This corresponds to the classic definition of an equilibrium, .

The second-order variation is

$$H^{(2)}[f_0] = [G, [G, H]](f_0)$$

The complete calculation of $H^{(2)}$ is then possible.

$$[G, H](f) = \int f\{g, E\}\mathrm{d}\Gamma$$

>From the expression of E, we get directly $\frac{\delta E'}{\delta f} = -\frac{Gm^2}{|\mathbf{q}-\mathbf{q}'|}$.

It leads to

$$\begin{aligned}\frac{\delta[G, H]}{\delta f} &= \{g, E\} + \int f'\left\{g', -\frac{Gm^2}{|\mathbf{q}-\mathbf{q}'|}\right\}\mathrm{d}\Gamma' \\ &= \{g, E\} + \int \frac{Gm^2}{|\mathbf{q}-\mathbf{q}'|}\{g', f'\}\mathrm{d}\Gamma' \end{aligned} \tag{4}$$

>From this, computing $H^{(2)}$ is easy :

$$\begin{aligned} H^{(2)}[f_0] &= -\int \left.\frac{\delta[G,H]}{\delta f}\right|_{f=f_0} \{g, f_0\}\mathrm{d}\Gamma \\ &= -\int \left(\{g,E\} + \int \frac{Gm^2}{|\mathbf{q}-\mathbf{q}'|}\{g', f_0'\}\mathrm{d}\Gamma'\right)\{g, f_0\}\mathrm{d}\Gamma \\ &= -\int \{g,E\}\{g,f_0\}\mathrm{d}\Gamma - Gm^2 \iint \frac{\{g,f_0\}\{g',f_0'\}}{|\mathbf{q}-\mathbf{q}'|}\mathrm{d}\Gamma\mathrm{d}\Gamma' \end{aligned} \quad (5)$$

This method is a very powerful one, since it allows us to obtain $H^{(2)}$ efficiently in a very general case. Most methods to compute energy variation due to a perturbation require the knowledge of the system's actual geometry, that is to say $f_0(\mathbf{r}, \mathbf{v})$ and $\psi(r)$, which are difficult to obtain knowing only $f_0(E, L^2)$ (except in some particular cases). With this method, we do not need to know the system's geometry to compute the energy variation.

3.2 Stability criterion

In the classic case of one particle influenced only by conservative forces, the stability of an equilibrium is directly linked to the sign of the second-order energy variation : if it is positive semi-definite, then the equilibrium is stable; otherwise it is unstable. In the previous part, we derived the second-order energy variation around an equilibrium state : its sign should provide a criterion to know whether it is stable or unstable. Unfortunately, in more general cases, this is not so simple.[5]

In the case where $H^{(2)} > 0$ for all generators g, then there is a definitive result by Bartholomew (1971), which proves that the system is *stable*. The symplectic method at least allows to prove the stability of an equilibrium. However, in the case where there are generators g such that $H^{(2)} < 0$, there is no definite proof of instability, at least not without more hypotheses.

An important mathematical result was given by Bloch, Krishnaprasad, Marsden, and Ratiu (1994), in a quite general case. Consider a Hamiltonian system with finite dimension, that is initially at equilibrium. Then we suppose there exists a negative energy mode. In this case, Bloch et al. proved that with the addition of dissipation, the equilibrium becomes spectrally unstable, from which follow linear and nonlinear instability. This kind of instability can be called a *dissipation-induced instability.*

As the Vlasov–Poisson is infinite-dimensional, this result doesn't directly apply to our problem. More recent works by Rouslan and Marsden (2009) on the infinite dimensional case seem to indicate that it works in the same way; there is no general proof yet, but a result seems likely in the near future.

The previous method allows us to retrieve previous results much more easily. In the isotropic case $f_0(E)$, this method proves stability against all perturbations provided that $\partial_E f_0 < 0$. In the anisotropic case $f_0(E, L^2)$, it proves (with the same condition $\partial_E f_0 < 0$) stability against all so-called preserving perturbations, *i.e.* perturbations that verify $\{g, L^2\} = 0$, see **?**.

[5] An example is the case of a charged particle in a negative harmonic potential, with a strong enough magnetic field. This example will be covered in more details in a future paper (Maréchal and Perez, 2009).

3.3 Perspectives for radial orbits

A system with nearly radial orbits can be easily described with the previous formalism, as $f_0(E, L^2) = \varphi(E)\delta(L^2)$ with δ a function that selects values near 0 (for instance a Dirac distribution). In this case, it is possible to show that for a sufficiently selective function, there are negative energy modes. The details are to be published in a future article (Maréchal and Perez, 2009, soon to be submitted).

The idea is that $H^{(2)}$ has two main components :

$$H^{(2)} = \underbrace{-\int \{g, E\}\{g, f\}\mathrm{d}\Gamma}_{(A)} \underbrace{-Gm^2 \iint \frac{\{g, f\}\{g', f'\}}{|\mathbf{q} - \mathbf{q}'|}\mathrm{d}\Gamma\mathrm{d}\Gamma'}_{(B)}$$

The term (A) mostly corresponds to kinetic energy variation, while (B) corresponds to potential energy variation. For a distribution function that is sufficiently radial, it is possible to show that there is a class of generators g breaking spherical symmetry, such that (A) is negligible in front of (B). The latter is negative, as the integral of a function against its own Laplacian. This proves the existence of negative energy modes.

4 Conclusion

We have shown a criterion to discuss the stability of Hamiltonian systems, and used it in the case of self-gravitating systems such as star clusters or galaxies, thanks to the the fact that Vlasov–Poisson is a Hamiltonian system. This criterion works on initial equilibrium states. The addition of dissipation seems to imply an instability if a steady state has negative energy modes. Apparently, systems populated with radial orbits have negative energy modes, which would mean they are unstable and susceptible to lose their spherical symmetry, triggering radial orbit instability.

It may be possible to give a physical interpretation of this result. Radial orbits have no tangential velocity, so they do not precess around the system center; instead they are confined on a line. Bringing the orbits closer is be possible, as they do not precess, and it leads to a lower energy state as the stars would be on average closer to each other than before. If a direction had a higher than average density, other orbits would tend to align in this direction, in a lower energy state; those orbits would bring on others, and so on, leading to an instability. This process would not be possible if the stars cannot dissipate energy; hence the necessity of dissipation to insure that radial orbit instability takes place.

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