Collision Avoidance by Optimal Control Techniques

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Contents

Motivation

Formulation of OCP

OC-CODE

Output

Sensitivity Analysis

Simulation
Goal: development of driver assistance systems that help to reduce severeness of accidents

Passive safety systems:
▶ chassis, airbags, seat belts, seat belt tighten, ...

Driver assistance systems in use:
▶ anti-blocking system (ABS), braking assistant (BAS), active brake assist in trucks (ABA)
▶ anti-slip regulation (ASR)
▶ electronic stability control (ESC, ESP, DSC, ...)
▶ adaptive cruise control (ACC)
▶ ...

Future driver assistance systems:
▶ collision avoidance, active steering, car-to-car communication, ...
Scenario 1: A stationary obstacle at a given distance to an approaching car, which drives at a prescribed speed, has to be avoided.

\[
\text{TimeToCollision} = \frac{(x_{\text{obstacle}} - x_{\text{car}}(t_0))}{v_x(t_0)} < 2\text{sec}.
\]

**Questions:** Can a collision be avoided at all? If yes, how?
Single-Track Model

Usage: basic investigation of lateral dynamics, controller design

\[
\ell_r \quad e_{SP} \quad \ell_f
\]

\[
F_{\ell r} \quad F_{sr} \quad (x, y)
\]

\[
F_{sf} \quad F_{\ell f}
\]
Car Model

Single track model:

- Simplifying assumption: rolling and pitching behavior of the car body can be neglected.

- Control variables:
  \[ w_{\delta,\text{min}} \leq w_{\delta} \leq w_{\delta,\text{max}} \]  
  (steering velocity)
  \[ F_{B,\text{min}} \leq F_{B} \leq F_{B,\text{max}} \]  
  (braking force)

- State variables:
  - car’s center of gravity: \((x, y)\)
  - yaw angle: \(\psi\)
  - yaw angle rate: \(w_{\psi}\)
  - velocities: \(v_x\) and \(v_y\) in \(x\)- and \(y\)-direction, respectively
  - steering angle: \(\delta\)

- Optimization parameters:
  - free final time: \(t_f\)
  - initial distance between \(x\)-coordinate of the car and the obstacle: \(d\)
Dynamics:

\[
x'' = \frac{(F_x \cos(\psi) - F_y \sin(\psi))}{m}
\]

\[
y'' = \frac{(F_x \sin(\psi) + F_y \cos(\psi))}{m}
\]

\[
\psi'' = \frac{(\ell_f F_{sf} \cos(\delta) - \ell_r F_{sr} + \ell_f F_{\ell f} \sin(\delta))}{I_{zz}}
\]

\[
\delta' = w_\delta
\]

$m, l_{zz}, \ell_f, \ell_r$ are constants and $F_x, F_y, F_{sf}, F_{sr}, F_{\ell f}$ are nonlinear functions of the state $(x, y, \psi, v_x, v_y, w_\psi, \delta)$ such that

\[
\|(F_{sf}, F_{\ell f})\| \leq F_{\text{max}, f}, \quad \|(F_{sr}, F_{\ell r})\| \leq F_{\text{max}, r} \quad \text{(Kamm's circle)}.
\]
This approach aims at reaching a safe target state and it is realized by the following optimal control problem OCP($y_{\text{car},0}, \nu_{x,0}$):

The **Objective Function** consists in the minimization of a linear combination of different factors, where $c_1, \ldots, c_4$ are suitable constants:

$$
\begin{align*}
\min & \ \ c_1 \ t_f & \text{(final time)} \\
+ & \ \ c_2 \ d & \text{(initial distance car-obstacle)} \\
+ & \ \ c_3 (y_{\text{target}} - y_{\text{car}}(t_f))^2 & \text{(distance to a safe target point)} \\
+ & \ \ c_4 \int_{t_0}^{t_f} w_{\delta}(t)^2 dt & \text{(steering effort)}
\end{align*}
$$

where:

$$
y_{\text{target}} = y_{\text{obstacle}} + \frac{\text{width}_{\text{obstacle}}}{2} + \frac{\text{width}_{\text{car}}}{2} + 0.3
$$
subject to the following constraints:

(a) differential equations: given by the dynamics (1)-(4)

(b) initial conditions:

\[(x(0), y(0), \psi(0), v_x(0), v_y(0), w_\psi(0), \delta(0)) = (0, y_{car,0}, 0, v_x, 0, 0, 0)\]

(c) state constraints:

\[1.3 \leq y(t) \leq 5.7 \quad \text{(stay on road)}\]

\[\| (F_{sv}, F_{uv}) \| \leq F_{\text{max},v}, \quad \| (F_{sh}, F_{uh}) \| \leq F_{\text{max},h} \quad \text{(Kamm's circle)}\]

(d) boundary conditions:

\[x(t_f) = d \quad \text{(d is a optimization parameter, if } c_2 \neq 0, \text{ or a constant)}\]

\[y'(t_f) = 0 \quad \text{(no velocity in y-direction when passing obstacle)}\]

(e) control constraints:

\[w_{\delta,\text{min}} \leq w_\delta \leq w_{\delta,\text{max}} \quad \text{(steering velocity)}\]

\[F_{B,\text{min}} \leq F_B \leq F_{B,\text{max}} \quad \text{(braking force)}\]
Software by M. Gerdts

Software **OC-ODE, OCPID-DAE1** [G.]:
- **direct multiple shooting** discretization
- **SQP method** (non-monotone linesearch, filter, BFGS update, primal active-set QP solver)
- various **integrators** (Runge-Kutta, BDF methods, linearized Runge-Kutta methods)
- various **control approximations** (B-splines of order $k$)
- sensitivity analysis
Direct Shooting

Minimize \( \Phi(x_h(t_0; z), x_h(t_N; z), w) \)

s.t. 
\[
\begin{align*}
    c(t_i, x_h(t_i; z), u_h(t_i; z), w) & \leq 0, \quad \forall i, \\
    s(t_i, x_h(t_i; z), w) & \leq 0, \quad \forall i, \\
    \psi(x_h(t_0; z), x_h(t_N; z), w) & = 0, \\
    x_h(\bar{t}_j; z) - x_j & = 0, \quad \forall j
\end{align*}
\]

Single shooting method: small & dense, with \( z = (x_1, u_1, \ldots, u_N) \).
The time interval \([t_0, t_f] \subset \mathbb{R}\) must be a non-empty and bounded interval with fixed time points \(t_0 < t_f\).

The objective function is the function:

\[
\min \Phi(x(t_0), x(t_f), t_f, w).
\]
Some simplifications

- In our case the problem has free final time $t_f$ and can be transformed to a problem with fixed final time by introducing the artificial time $\tau \in [0, 1]$ by

$$t(\tau) := t_0 + \tau(t_f - t_0)$$

The differential equations are:

$$0 = F(t(\tau), \bar{x}(\tau), \bar{x}'(\tau)/(t_f - t_0), \bar{u}(\tau), w).$$

The free final time $t_f$ is considered as an optimization parameter.

- Moreover the objective function related to our scenario is of the form:

$$J[x, u, p] = \Phi(x(t_0), x(t_f), t_f, w) + \int_{t_0}^{t_f} f_0(t, x(t), u(t), w) dt$$

and it can be transformed by introducing an additional differential equation

$$x'_0(t) := f_0(t, x(t), u(t), w), x_0(t_0) = 0 \Rightarrow$$

$$\bar{x} := (x_0, x)^\top, \bar{J}[\bar{x}, u, w] := \Phi(x(t_0), x(t_f), t_f, w) + x_0(t_f).$$
Trajectory for several objective functions

\[
\min \int_{t_0}^{t_f} w_\delta(t)^2 \, dt \\
\quad c = (0, 0, 0, 1)
\]

\[
\min (y_{\text{target}} - y_{\text{car}(t_f)})^2 \\
\quad c = (0, 0, 1, 0)
\]

\[
\min d \\
\quad c = (0, 1, 0, 0)
\]

\[
\min t_f \\
\quad c = (1, 0, 0, 0)
\]

Data: \( y_{\text{car}, t_f} - y_{\text{target}} = 2.1 \, \text{m}, y_{\text{obstacle}, 0} = y_{\text{car}, 0} = 1.75 \, \text{m}, v_y, 0 \leq 0 \, \text{m/s}, v_x, 0 = 45 \, \text{m/s}, d \leq 45 \, \text{m}, \text{CPU: 0.05 s - 0.07 s} \)
Objective function: $\min 10t_f + 0.1d + \int_0^{t_f} w_\delta(t)^2 dt$, i.e. $c = (10, 0.1, 0, 1)$

$y_{\text{car},0} = 2.44m$, $v_{x,0} = 20m/s$ $y_{\text{car},0} = 3.14m$, $v_{x,0} = 38m/s$ $y_{\text{car},0} = 1.75m$, $v_{x,0} = 50m/s$

$t_f = 0.97s$, $d = 19.46m$ $t_f = 0.75s$, $d = 27.16m$ $t_f = 1.13s$, $d = 53.15m$

Data: car and obstacle width 1.8 m, car and obstacle length 3.5 m, road width 7 m, initial y-position of the obstacle 1.75 m, initial y-velocity of the car 0 m/s, $y_{\text{car},t_f} - y_{\text{target}} = 2.1 \text{ m}$, CPU: 0.05 s - 0.07 s
Parametrized OCP

Sensitivity analysis with respect to initial values:

\[ \min c_1 t_f + c_2 d + c_3 (y_{\text{target}} - y_{\text{car}}(t_f))^2 + c_4 \int_{t_0}^{t_f} w_\delta(t)^2 \, dt \]

(a) differential equations: given by the dynamics (1)-(4)
(b) initial conditions:
\[ (x(0), y(0), \psi(0), v_x(0), v_y(0), w_{\psi}(0), \delta(0)) = (p_1, p_2, p_3, p_4, p_5, 0, 0) \]

(c) state constraints:
\[
1.3 \leq y(t) \leq 5.7 \quad \text{(stay on road)}
\]
\[
\| (F_{sv}, F_{uv}) \| \leq F_{\text{max},v}, \quad \| (F_{sh}, F_{uh}) \| \leq F_{\text{max},h} \quad \text{(Kamm's circle)}
\]
(d) boundary conditions:
\[ x(t_f) = d \quad (d = \text{initial distance to obstacle}) \]
\[ y'(t_f) = 0 \quad \text{(no velocity in y-direction when passing obstacle)} \]

(e) control constraints:
\[ w_{\delta,\text{min}} \leq w_\delta \leq w_{\delta,\text{max}} \quad \text{(steering velocity)} \]
\[ F_{B,\text{min}} \leq F_B \leq F_{B,\text{max}} \quad \text{(braking force)} \]
Sensitivity analysis

Investigate the dependence of the optimal solution $\hat{x}(p_0)$ (continuously differentiable with respect to $p$), with respect to parameter $p$:

$$\hat{x}(p) \approx \hat{x}(p_0) + \frac{d\hat{x}}{dp}(p_0)(p - p_0).$$  \quad (5)

- We then need to investigate the sensitivity $\frac{d\hat{x}}{dp}(p_0)$ computing it by linearization via Sensitivity Theorem.
- No complete discretization of the states, so we need to split the computation of this differential in two stages via chain rule:

$$\frac{d\hat{x}}{dp} = \frac{d\hat{x}}{dz} \frac{dz}{dp};$$  \quad (6)

where $z$ is the vector of the optimization variables (states, controls,..).
Two steps for sensitivity analysis

First step:
Fix the optimal control $\hat{u}$ and investigate the dependence of the state $\hat{x}$ with respect to the optimization vector $z(p)$:

$$S(t) := \frac{\partial \hat{x}(t; z(p))}{\partial z(p)}, \quad t \in [t_0, t_f].$$

Solution: Solve the sensitivity differential equation

$$S'(t) = f_x'(\hat{x}(t; z(p)), \hat{u}(t; z(p)))S(t) + f_u'(\hat{x}(t; z(p)), \hat{u}(t; z(p)))\frac{\partial \hat{u}(t; z(p))}{\partial z(p)}$$
$$S(t_0) = x_0'(z(p)).$$

Second step:
Compute $z'(p)$ using the following Sensitivity Theorem.
Parametric optimization

**NLP(\(p\))**

Minimize \(f(z, p)\)

s.t. \(g_i(z, p) = 0, \quad i = 1, \ldots, n_E,\)

\(g_i(z, p) \leq 0, \quad i = n_E + 1, \ldots, n_g\)

**Sensitivity [Fiacco’83]**

Let \(\hat{z}\) be a strongly regular local minimum of \(NLP(p_0)\) for nominal parameter \(p_0\). Then:

- \(NLP(p)\) has unique strongly regular local minimum \(z(p), \lambda(p)\) near \(p_0\).
- \(z(p), \lambda(p)\) are continuously differentiable w.r.t. \(p\).
Second step

Moreover

- nominal parameter $p_0$
- strongly regular local solution $z(p_0)$ to $NLP(p_0)$
- sensitivities $\frac{dz}{dp}(p_0)$: At $p = p_0$ it holds

\[
\begin{pmatrix}
\frac{d\hat{z}}{dp} \\
\frac{d\hat{\lambda}_{A(\hat{z},p_0)}}{dp}
\end{pmatrix}
= - \begin{pmatrix}
\nabla_{zz}^2 L & (\nabla z g_A(\hat{z},p_0))^T \\
\nabla z g_A(\hat{z},p_0) & 0
\end{pmatrix}^{-1}
\begin{pmatrix}
\nabla_{zp}^2 L \\
\nabla p g_A(\hat{z},p_0)
\end{pmatrix}
\]
Optimal solution with objective function $\min t_f + d + \int_{t_0}^{t_f} w_\delta(t)^2 dt$ ($c = (1, 1, 0, 1)$) and initial data $y_{\text{car}, t_f} - y_{\text{target}} = 2.1 m$, $y_{\text{obstacle}, 0} = y_{\text{car}, 0} = 1.75 m$, $v_y, 0 \leq 0 m/s$, $v_x, 0 = 45 m/s$, $d \leq 45 m$

Perturbed Trajectories

perturbation of $\varepsilon = -0.1 m$ in the initial states $(x(0), y(0), \psi(0), v_x(0), v_y(0))$

perturbation of $\varepsilon = +0.1 m$ in the initial states $(x(0), y(0), \psi(0), v_x(0), v_y(0))$
Here we have the non-perturbed trajectory (black curve) with the trajectories obtained by perturbing the initial data \((x(0), y(0), \psi(0), v_x(0), v_y(0))\) of a factor \(\pm 0.1\).
Sensor tolerances

- **Specification of sensor accuracy:** For a given $\varepsilon > 0$ find $\delta > 0$ with

  $$\|p - p_0\| < \delta \implies \|x(t_f; z(p)) - x(t_f; z(p_0))\| < \varepsilon.$$ 

- **Approach:**

  $$x(t_f; z(p)) - x(t_f; z(p_0)) \approx \left( \frac{dx}{dp}(t_f; z(p_0)) \right) \cdot (p - p_0) \implies$$

  $$\left\| \left( \frac{dx}{dp}(t_f; z(p_0)) \right)^{-1} \right\| \cdot \|x(t_f; z(p)) - x(t_f; z(p_0))\| \geq \|p - p_0\|$$

  for suitable norms $\| \cdot \|$. 
Real World Application!!!
Lego-Robot model

Let $t_f = \text{final time}$ and $d = \text{initial distance between car and obstacle}$:

$$\min t_f + d$$

subject to the following constraints:

(a) differential equations:

$$x' = \frac{v_l + v_r}{2} \cos(\psi), \quad y' = \frac{v_l + v_r}{2} \sin(\psi)$$

$$v'_l = u_l, \quad v'_r = u_r$$

(b) initial conditions:

$$(x(0), y(0), \psi(0), v_l(0), v_r(0)) = (0, 0, 0, 0, 0)$$

(c) state constraints:

$$(d − x)^2 + (0 − y)^2 \geq (\text{radius}_{\text{car}} + \text{radius}_{\text{obstacle}})^2$$

(d) boundary conditions:

$$x(t_f) = d \quad \text{(stop after passing the obstacle)}$$

$$\psi(t_f) = 0 \quad \text{(the final direction has to be the x-axle)}$$

$$v_l(t_f) = 0 = v_r(t_f) \quad \text{(stop at } t_f\text{)}$$

(e) control constraints:

$$-0.4 \leq u_l \leq 0.4 \quad \text{(left wheel acceleration)}$$

$$-0.4 \leq u_r \leq 0.4 \quad \text{(right wheel acceleration)}$$
Lego-Robot simulation

- Run the OCP with OC-CODE and get the output with all the controls value at each time step.
- Simulation (developed with Martin Kunkel-Uni BW):
Thanks for your attention!

▶ Questions?
▶ Further Information: ilaria.xausa@unibw.de