Internship proposal: numerical studies of the stability of the time-domain FEM-BEM coupling

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1 Context

Many applications (radar detection, underwater acoustics, musical acoustics) require modelling of the time domain wave propagation in unbounded domains. For the numerical treatment of such problems it is necessary to be able to bound the computational domain. This is done with the help of so-called transparent boundary conditions. One way to realize them is to use the formalism of time-domain boundary integral equations.

A classical method of incorporating mathematically the transparent boundary conditions into the formulation is provided by the use of the Dirichlet-to-Neumann (DtN) map. This DtN map can be expressed in terms of boundary integral operators. Thus, the original problem (in the unbounded domain) can be rewritten as a coupled system of PDEs inside the (bounded) computational domain and boundary integral equations on the boundary of the computational domain.

Let us remark that the corresponding coupling is called non-symmetric; it provides a less expensive (in terms of the computational costs) alternative to the use of the symmetric coupling [1,2].

There are multiple ways to discretize the respective formulations. For the volumic part of the formulation one often uses time-stepping methods coupled with the finite element methods for spatial discretization. As for the boundary integral equations, one could use either the convolution quadrature [3] method for the time discretization combined with the Galerkin boundary elements in space, or space-time Galerkin methods. This list of possible discretizations is of course non-exhaustive.

Nonetheless, while on the continuous level stability of the respective formulation is rather clear, this question appears to be non-trivial for the discretized formulation [4,5].
2 Description

The goal of this project is to validate numerically the stability of the discretizations of the non-symmetric coupling. In particular, this project consists of several stages:

- formulate the discretized formulation for the FEM/BEM coupling. For this we will use the Galerkin method in space and the convolution quadrature method in time (in the spirit of [6])
- implement the algorithm using the XLife++ library
- perform various numerical experiments to verify the stability of the coupling

3 Future developments

This subject may be developed into a PhD thesis. Depending on the outcome of the numerical experiments, one of the possibilities is to attempt to prove the stability of the coupling. Another possibility is to delve further into studies of the time-domain boundary equations, in particular, by enhancing the error analysis of their space-time Galerkin discretizations based on the recently obtained improved stability estimates of time-domain boundary integral operators [7, 8].

4 Desired skills

We are looking for a candidate with a strong background in numerical analysis and programming skills. Background in complex analysis is desirable but not necessary.

5 References

4. V. Gruhne, Numerische Behandlung zeitabhängiger akustischer Streuung im Außen- und Freiraum, PhD thesis 2013, University of Leipzig

6. P. Joly, M. Kachanovska, Transparent boundary conditions for wave propagation in fractal trees: convolution quadrature approach, https://hal.archives-ouvertes.fr/hal-02265345/document
