Invisible deformation of the boundary of an acoustic waveguide

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Keywords: waveguide, scattering matrix, invisibility, asymptotic theory, fixed point

The question of whether a scatterer can be invisible, in the sense that it does not scatter incident waves, is currently the subject of an active research. Here we consider a two-dimensional homogeneous acoustic waveguide and we aim at designing deformations of the boundary which are invisible at a given frequency, or more generally at a finite number of given frequencies. To find such invisible perturbations, we take advantage of the fact that there are only a finite number of propagative modes at a given frequency in a waveguide. As a consequence, the invisibility is achieved by canceling a finite number of scattering coefficients, and an invisible deformation only produces an exponentially decreasing scattered field, not measurable in the far field.

The first step consists in studying the effect of a small deformation, of amplitude ε. The asymptotic analysis allows to derive the first order terms of the scattering coefficients, as integrals involving the function describing the deformation. For instance, les us consider a waveguide of height 1 and a wavenumber $k < \pi$, so that only the plane mode is propagating. Then a small deformation of the upper boundary given by $y = \varepsilon h(x)$ produces a reflection of the plane mode, with a reflection coefficient R_ε such that

$$R_\varepsilon = ik\varepsilon \int h(x)e^{2ikx}dx + O(\varepsilon^2).$$

This leads, following [1] (where the approach of [3] is used), to express the deformation h as a linear combination of some explicit (compactly supported) functions, so that the condition $R_\varepsilon = 0$ is satisfied if and only if the coefficients of the linear combination are solution of a fixed point equation. The key point is that we can prove, using the results of the asymptotic analysis, that the function of this fixed point equation is a contraction for ε small enough. This proves the existence of invisible deformations of amplitude ε (only a phase shift of order ε^2 remains that we are not able to remove). Moreover, it provides a natural algorithm to compute the invisible deformation.

This has been tested numerically. At each iteration of the fixed-point algorithm, we have to compute R_ε for a given h. To avoid remeshing, we use a multimodal method as in [2]. The results are in perfect agreement with the theory. The good news is that ε can be taken quite large (the amplitude of the deformation may be half the size of the guide). We are able to build both invisible bumps and invisible cavities. Let us point out that an invisible cavity leads by symmetry to an invisible obstacle in the middle of a guide two times larger.

References

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