

Extraordinary transmission through subwavelength dielectric gratings in the microwave range

Ahmed Akarid,¹ Abdelwaheb Ourir,^{1,*} Agnès Maurel,¹ Simon Félix,² and Jean-François Mercier³

¹Institut Langevin, CNRS, ESPCI ParisTech, 1 rue Jussieu, 75005 Paris, France

²LAUM, CNRS, Université du Maine, avenue Olivier Messiaen, 72085 Le Mans, France

³Poems, CNRS, ENSTA ParisTech, INRIA, 828 boulevard des Maréchaux, 91762 Palaiseau, France

*Corresponding author: a.ourir@espci.fr

Received December 20, 2013; revised May 13, 2014; accepted May 18, 2014;

posted May 19, 2014 (Doc. ID 203529); published June 18, 2014

We address the problem of the transmission through subwavelength dielectric gratings. Following Maurel *et al.* [Phys. Rev. B **88**, 115416 (2013)], the problem is reduced to the transmission by an homogeneous slab, either anisotropic (for transverse magnetic waves, TM) or isotropic (for transverse electric waves, TE), and an explicit expression of the transmission coefficient is derived. The optimum angle realizing perfect impedance matching (Brewster angle) is shown to depend on the contrasts of the dielectric layers with respect to the air. Besides, we show that the Fabry–Perot resonances may be dependent on the incident angle, in addition to the dependence on the frequency. These facts depart from the case of metallic gratings usually considered; they are confirmed experimentally both for TE and TM waves in the microwave regime. © 2014 Optical Society of America

OCIS codes: (160.3918) Metamaterials; (230.4555) Coupled resonators.

<http://dx.doi.org/10.1364/OL.39.003752>

Recently, the nonresonant extraordinary transmission (EOT) through subwavelength metallic gratings has been reported in the context of electromagnetic transverse magnetic (TM) waves [1–4] and in the context of acoustic waves [5–7]. Varying the incident angle and the frequency, the transmission spectra were shown to present two main features that depart from the vanishing transmission expected for such rigid structures: (1) Fabry–Perot resonances (FP) occurring at given frequencies, for all incident angles, and (2) a perfect transmission, independent of the frequency, occurring at a prescribed incident angle, referred as the Brewster angle. In most studies, the Brewster angle was given by the filling fraction of the metal in the grating. In a recent paper [8], the transmission spectrum of a grating made of alternating layers of a dielectric material and air was studied, revealing the possible dependence of the Brewster angle on the material properties and on the microstructure of the grating, the case of metallic grating being a limiting case. Such study may open the way for the design of gratings with controlled transmission properties.

In this Letter, we generalize the results of [8] to gratings made of alternating layers of two dielectric materials. As in [8], homogenization theory is used to derive the effective properties of the grating, and, for both TM and TE waves, its transmission coefficient. It is shown that, as in the metallic case, EOT is observable only for TM waves, and the dependence of the Brewster angle on the geometry and material properties is given. For both polarizations, the FP resonances are shown to be dependent (in general) on the incident angle, in addition to the usual dependence on frequency. An experimental study in the microwave regime is presented, for both polarizations and two different gratings. Results on the transmission spectra are shown to present an excellent agreement with the theory.

We consider a grating composed of alternating layers of two nonmagnetic dielectric materials (with relative

permeability equal to unity) and relative permittivities ϵ_1 and ϵ_2 (Fig. 1). The layers have widths w_1 and w_2 , respectively, and the total length of the grating is denoted ℓ . The grating is placed in air and illuminated with an incident wave with wavenumber $k = 2\pi f/c$, with f the frequency and c the speed of light in air, and it is assumed that $kw_1, kw_2 \ll 1$. To begin with, we consider a TM incident wave. Thus, the magnetic field $\mathbf{H} = H(\mathbf{r})\hat{\mathbf{z}}$, with $\mathbf{r} = (x, y)$, satisfies

$$\nabla \cdot \left(\frac{1}{\epsilon(\mathbf{r})} \nabla H(\mathbf{r}) \right) + k^2 H(\mathbf{r}) = 0, \quad (1)$$

and $\epsilon(\mathbf{r}) = \epsilon(y) = \epsilon_{1,2}$ inside the grating. The grating is a subwavelength layered structure that can be described as an anisotropic homogeneous media, using the homogenization theory of layered media [8]. This leads to

$$\nabla \cdot \left[\begin{pmatrix} 1/\epsilon_{\parallel} & 0 \\ 0 & 1/\epsilon_{\perp} \end{pmatrix} \nabla H \right] + k^2 H = 0, \quad (2)$$

with

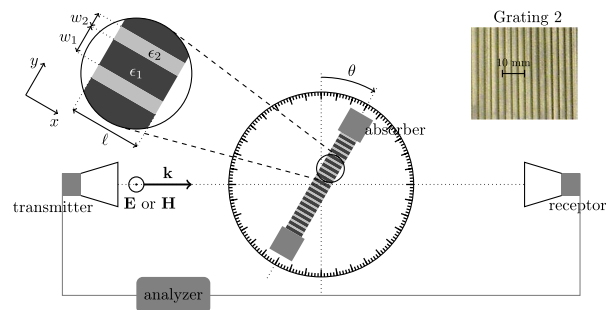


Fig. 1. Schematic view of the experimental setup; the two insets show the geometry of the dielectric grating and an image of the grating 1 (epoxy/air).

$$\begin{cases} \frac{1}{\epsilon_{\parallel}} = \frac{w_1}{(w_1+w_2)} \frac{1}{\epsilon_1} + \frac{w_2}{(w_1+w_2)} \frac{1}{\epsilon_2}, \\ \epsilon_{\perp} = \frac{w_1}{(w_1+w_2)} \epsilon_1 + \frac{w_2}{(w_1+w_2)} \epsilon_2. \end{cases} \quad (3)$$

In the case of TE waves ($\mathbf{E} = E(\mathbf{r})\hat{\mathbf{z}}$), we have

$$\nabla \cdot \nabla E(\mathbf{r}) + k^2 \epsilon(\mathbf{r})E(\mathbf{r}) = 0, \quad (4)$$

from which the homogenization gives an equivalent isotropic medium

$$\nabla \cdot \nabla E + k^2 \epsilon_{\text{eff}} E = 0, \quad \text{with } \epsilon_{\text{eff}} = \epsilon_{\perp}. \quad (5)$$

In both cases, the transmission coefficient in amplitude t through such structure, when embedded in air (with relative permittivity $\epsilon_0 \simeq 1$) is simply

$$t = \frac{4Z_r e^{ik\ell} \cos \theta}{(1 + Z_r)^2 e^{-ik\ell} - (1 - Z_r)^2 e^{ik\ell}}, \quad (6)$$

where k_{\parallel} is the effective wavenumber along the x -direction in the homogenized structure ($k \sin \theta$ being the wavenumber component along the y -direction) and Z_r the relative impedance of the homogenized grating with respect to the air. These are given by

$$\begin{cases} \frac{k_{\parallel}^2}{\epsilon_{\parallel}} + \frac{k_{\perp}^2}{\epsilon_{\perp}} \sin^2 \theta = k^2, & Z_r = \frac{k}{k_{\parallel}} \epsilon_{\parallel} \cos \theta, \quad \text{for TM waves,} \\ k_{\parallel}^2 + k_{\perp}^2 \sin^2 \theta = k^2 \epsilon_{\text{eff}}, & Z_r = \frac{k}{k_{\parallel}} \cos \theta, \quad \text{for TE waves.} \end{cases} \quad (7)$$

The simple expression (6) gives the standard FP resonances for $e^{ik_{\parallel}\ell} = \pm 1$ and the condition of perfect transmission when the matched impedance condition is realized: $Z_r = 1$. According to Eqs. (7), the FP resonances occur at a frequency that is angular sensitive, and the matching impedance condition, when realized, is insensitive to frequency. This is detailed further below, when compared to the experimental measurements.

We performed transmission measurements for two gratings in the microwave X-band (8 to 12 GHz) range (Fig. 1). The gratings have been fabricated using alternating layers of widths $w_1 = 2$ mm and $w_2 = 1$ mm and total length $\ell = 30$ mm. The first grating, hereafter named grating 1, is composed of epoxy and air layers with permittivities $\epsilon_1 = 4.4$ and $\epsilon_2 = 1$, respectively. The second one (grating 2) is composed of epoxy and teflon layers with permittivities $\epsilon_1 = 4.4$ and $\epsilon_2 = 2.2$, respectively. In the two transverse directions, the gratings are roughly 300×100 mm (along y and z , respectively). At our working frequencies, the wavelength in air is typically between 2.5 and 4 cm, well above the typical size of the grating microstructure, w_1 and w_2 . In order to check the theoretical predictions on the spectra, transmission experiments have been realized, varying the frequency and the incident angle. To do that, the gratings have been placed on a rotational stage, realizing incident angles from 0° to 70° , each 2° (higher angles are not accessible to the measurements due to the limited transverse size of the structures). The experiments have been performed in a semi-anechoic chamber using a network analyzer and two horn antennas working as transmitter and

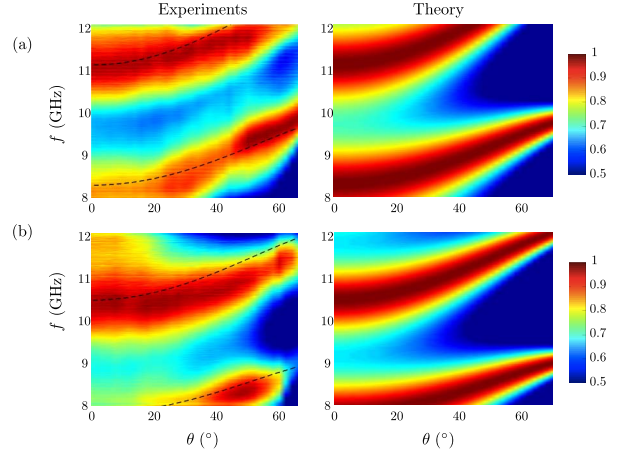


Fig. 2. Transmission spectra T^{exp} experimentally measured (left) and $T = |t|^2$ from Eq. (6) with Eq. (7) (right), as a function of the incident angle θ and of the frequency f for incident TE waves. (a) Grating 1 (epoxy-air) and (b) grating 2 (epoxy-teflon). Dotted lines show the theoretical positions of the Fabry-Perot resonances, Eq. (8), with $m = 3$ and 4.

receptor. Measurements of the transmission spectra have been made for 401 frequency values between 8 and 12 GHz (frequency increment of 10 MHz).

Figure 2 shows the transmission spectra T^{exp} measured for the two gratings and incident spectra TE waves. Corresponding spectra obtained from our theoretical predictions, $T = |t|^2$ in Eqs. (6) and (7), are shown for comparison. The agreement between our predictions and the measured spectra is very good in both cases. To be more quantitative, we measured the error $E = \langle |T^{\text{exp}} - T| \rangle / \Delta T$. Here, $\Delta T = \max(T) - \min(T)$ is used rather than the mean value of T since we would obtain an artificially low error when T remains within values close to unity. $\langle \cdot \rangle$ denotes the mean value, averaged over the whole spectra. We get errors of 11.4% for grating 1 and 15.5% for grating 2.

The dependence on incident angle of the FP resonant frequencies, f_{FP} , is confirmed and quantitatively recovered: dotted lines indicate the FP prediction given by

$$f_{\text{FP}} = m \frac{c}{2\ell} (\epsilon_{\text{eff}} - \sin^2 \theta)^{-1/2}, \quad (8)$$

(with m an integer) corresponding to $k_{\parallel}\ell = m\pi$ in Eq. (7). In Fig. 2, FP resonances corresponding to $m = 3$ and 4 are visible. According to this formula, the f_{FP} increases with the increase of θ and decreases with the increase of ϵ_{eff} ($\epsilon_{\text{eff}} = 3.3$ for the grating 1, and $\epsilon_{\text{eff}} = 3.7$ for the grating 2). Also, the frequency shift Δf between two FP resonances ($\Delta k_{\parallel} = \pi$) increases with θ and decreases with ϵ_{eff} . At $\theta = 40^\circ$, we get: for grating 1, $f_{\text{FP}} = 8.9$ and 11.6 GHz from the measurements, thus $\Delta f = 2.7$ GHz [theoretically, $f_{\text{FP}} = 8.8$ GHz ($m = 3$) and 11.8 GHz ($m = 4$)]. For grating 2, $f_{\text{FP}} = 8.2$ GHz and 10.6 GHz from the measurements, thus $\Delta f = 2.4$ GHz [theoretically, $f_{\text{FP}} = 8.3$ GHz ($m = 3$) and 11 GHz ($m = 4$)].

It results in a possible tuning of the angular sensitive transparency of the realized grating. Similar tuning of the FP resonance frequencies has been reported for metallic

gratings illuminated by TE waves [9]. In this case, the metallic grating was coupled to a dielectric cover layer with metallic wire array, whose geometrical properties were used to tune f_{FP} .

The absence of Brewster angle is also confirmed, as already reported in the case of metallic gratings. This is because the homogenized structure has a relative impedance, from Eq. (7),

$$Z_r = \frac{\cos \theta}{\sqrt{\epsilon_{\text{eff}} - \sin^2 \theta}}, \quad (9)$$

that can never match the impedance of the free space, equal to unity (indeed, since $\epsilon_{\text{eff}} > 1$, we have $|Z_r| \leq 1/\sqrt{\epsilon_{\text{eff}}} < 1$).

Similar comparisons are reported in Fig. 3 in the case of TM waves. The transmission being higher than in the TE case, the measurements appear noisier. However, the agreement remains very good and confirms the validity of our homogenized prediction: we get discrepancies of 6% for the grating 1 and 21% for the grating 2.

The FP frequencies, f_{FP} , depend on θ , and now they satisfy, from Eq. (7),

$$f_{\text{FP}} = m \frac{c}{2\ell} \frac{1}{\sqrt{\epsilon_{\parallel}}} \left(1 - \frac{\sin^2 \theta}{\epsilon_{\parallel}} \right)^{-1/2}. \quad (10)$$

Inspecting the expressions of ϵ_{\parallel} and ϵ_{\perp} in Eq. (3), it appears that a change in ϵ_2 produces a larger change in ϵ_{\parallel} than in ϵ_{\perp} : $\epsilon_{\parallel} = 2.06$ for the grating 1 and $\epsilon_{\parallel} = 3.3$ for the grating 2 (and $\epsilon_{\perp} = 3.3$ and 3.7, respectively). It follows that an increase in ϵ_2 produces a decrease in the f_{FP} : for $m = 3$, f_{FP} decreases from 10.4 GHz (grating 1) to 8.2 GHz (grating 2) at $\theta = 0$, and for $m = 4$, from 14 GHz [not visible on Fig. 3(a)] to 11 GHz. This is accompanied by a decrease in the distance between two successive FP. Similar tuning of the position in the f_{FP} and of the distance between two FP have been proposed in the case of metallic grating by changing the geometry of the metallic layers [10–14].

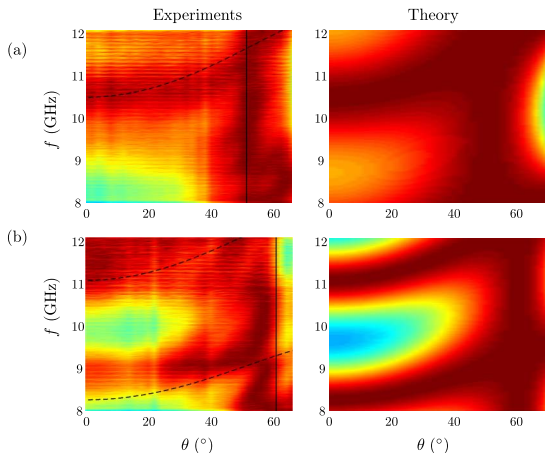


Fig. 3. Same representation as in Fig. 2, for incident TM waves. Dotted lines show the theoretical positions of the Fabry–Perot resonances, Eq. (10), here $m = 3$ for (a) and $m = 4$ for (b). Solid lines show the theoretical positions of the Brewster angles, Eq. (12).

The sensitivity of the Brewster angle on the contrasts is also confirmed. This is because the relative impedance of the structure is higher than for TE waves; from Eq. (7), we get

$$Z_r = \frac{\cos \theta}{\sqrt{\epsilon_{\perp} - \sin^2 \theta}} \sqrt{\epsilon_{\perp} \epsilon_{\parallel}}. \quad (11)$$

Thus, $|Z_r| \leq \sqrt{\epsilon_{\parallel}}$ can fulfill the matching impedance condition $Z_r = 1$ (since $\epsilon_{\parallel} > 1$) for

$$\theta^{\text{opt}} = \cos^{-1} \sqrt{\frac{\epsilon_{\perp} - 1}{\epsilon_{\perp} \epsilon_{\parallel} - 1}}, \quad (12)$$

which is always defined. From the above expression, an increase in ϵ_2 produces an increase in θ^{opt} : from $\theta^{\text{opt}} = 51.06^\circ$ (grating 1) to $\theta^{\text{opt}} = 60.65^\circ$ (grating 2) (plain lines in Fig. 3).

These behaviors contrast with the observations reported for metallic gratings, and they are briefly given in the following for comparison. Homogenization of metallic structures for TM waves is known to be more involved [15]. The case of metallic gratings has been considered in [16,17] in the electromagnetic case; alternatively, in [8], we consider the analog acoustic case in the Neumann limit, with both ϵ and $1/\mu$ (analog of the mass density ρ and of the bulk modulus $B = (\rho c^2)$, respectively) going to infinity. To compare with our two gratings, we suppose the epoxy in gratings 1 and 2 is replaced by a perfectly conducting metal. The homogenized structure satisfies (details can be found in [8]) (1) $k_{\parallel} \simeq k\sqrt{\epsilon_2}$ (the wavenumber in the dielectric material), and (2)

$$Z_r = \cos \theta \sqrt{\epsilon_2} \frac{w_2 + w_1}{w_2}, \quad (13)$$

from which the transmission coefficient can be calculated using Eq. (6). The corresponding spectra in the (θ, f) space are reported in Fig. 4. Note that our homogenized result agrees with [5] (note also a nonclassical homogenization proposed in [4], which makes the comparison difficult). We find the optimum angle to occur at $\theta^{\text{opt}} = \cos^{-1}[w_2/(w_1 + w_2)]/\sqrt{\epsilon_2}$, as reported numerically in [5] and experimentally (for $\epsilon_2 = 1$) in [3,6,7]. In our gratings 1 and 2, replacing the epoxy by metal, this

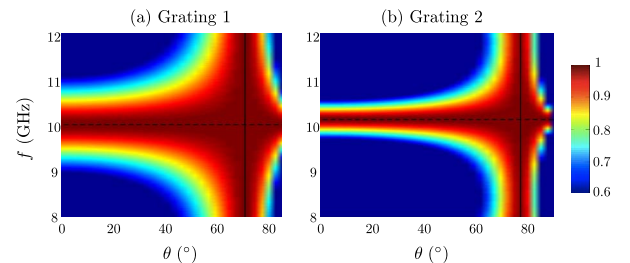


Fig. 4. Transmission spectra $|t|^2$ as a function of the incident angle θ and of the frequency f for TM waves (a) in a metal-air grating and (b) in a metal-tylon grating. Dotted lines show the positions of the Fabry–Perot resonances $k_{\parallel} \ell = m\pi$, here for (a) $m = 2$ and (b) $m = 3$. Plain lines show the positions of the Brewster angle.

would lead to optimum angles of 70.5° and 77° , for $\epsilon_2 = 1$ and $\epsilon_2 = 2.2$, respectively. A single FP is visible in the considered frequency range corresponding to $f_{\text{FP}} = mc/(2\ell\sqrt{\epsilon_2})$, here for $m = 2$ $f_{\text{FP}} = 10$ GHz and for $m = 3$ $f_{\text{FP}} = 10.11$ GHz, respectively. Because $\epsilon_{\perp} = \infty$, k_{\parallel} is independent of θ , these FP resonances are independent of θ .

In summary, we have shown that the transmission properties of gratings present angular and frequency sensitive transparencies, that are accurately described by homogenization theory. According to the resulting expression of the transmittance, the Brewster angle can be significantly decreased and tuned by considering two dielectric structures in the grating, while it is near the grazing angle in the case of metallic structures. Also, the FP resonances can be tuned, in position and relative frequency distance.

This work is supported by the Agence Nationale de la Recherche through the grant ANR ProCoMedia, project ANR-10-INTB-0914. A.O. and A.M. acknowledge the financial support of the LABEX WIFI (Laboratory of Excellence within the French Program "Investments for the Future") under references ANR-10-LABX-24 and ANR-10-IDEX-0001-02 PSL*.

References

1. X.-R. Huang, R.-W. Peng, and R.-H. Fan, *Phys. Rev. Lett.* **105**, 243901 (2010).
2. A. Alù, G. D'Aguanno, N. Mattiucci, and M. J. Bloemer, *Phys. Rev. Lett.* **106**, 123902 (2011).
3. N. Aközbeke, N. Mattiucci, D. de Ceglia, R. Trimm, A. Alù, G. D'Aguanno, M. A. Vincenti, M. Scalora, and M. J. Bloemer, *Phys. Rev. B* **85**, 205430 (2012).
4. C. Argyropoulos, G. D. Aguanno, N. Mattiucci, N. Aköztek, M. J. Bloemer, and A. Alù, *Phys. Rev. B* **85**, 024304 (2012).
5. C. Qiu, R. Hao, F. Li, S. Xu, and Z. Liu, *Appl. Phys. Lett.* **100**, 191908 (2012).
6. D.-X. Qi, R.-H. Fan, R.-W. Peng, S. R. Huang, M.-H. Lu, X. Ni, Q. Hu, and M. Wang, *Appl. Phys. Lett.* **101**, 061912 (2012).
7. G. D. Aguanno, K. Q. Le, R. Trimm, A. Alù, N. Mattiucci, A. D. Mathias, N. Aközbeke, and M. J. Bloemer, *Sci. Rep.* **2**, 340 (2012).
8. A. Maurel, S. Félix, and J.-F. Mercier, *Phys. Rev. B* **88**, 115416 (2013).
9. Y. Q. Ye and Y. Jin, *Phys. Rev. E* **80**, 036606 (2009).
10. M. J. Lockyear, A. P. Hibbins, and J. R. Sambles, *Appl. Phys. Lett.* **91**, 251106 (2007).
11. Y. Wang, Y. Wang, Y. Zhang, and S. Liu, *Opt. Express* **17**, 5014 (2009).
12. Y. Wang, J. Ye, Y. Zhang, and S. Liu, *Opt. Commun.* **284**, 877 (2011).
13. Z. Liu, H. Li, S. Xie, H. Xu, S. Fu, X. Zhou, and C. Wu, *Opt. Express* **17**, 5014 (2011).
14. C. Qiu, S. Li, R. Chen, B. Hou, F. Li, and Z. Liu, *Phys. Rev. B* **87**, 205129 (2013).
15. D. Felbacq, *J. Math. Phys.* **43**, 52 (2002).
16. J. Shin, J. Shen, P. B. Catrysse, and S. Fan, *IEEE J. Quantum Electron.* **12**, 1116 (2006).
17. J. T. Shen, P. B. Catrysse, and S. Fan, *Phys. Rev. Lett.* **94**, 197401 (2005).